A Circuit Parameter Extraction Algorithm of Eddy-Current Magnetic Field Based on Circuit-Field Coupling Method

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Abstract — A method to accurately extract the equivalent circuit parameters of eddy-current magnetic field is presented. Based on the solutions of the eddy-current magnetic field and electric circuit coupled model, the lumped parameters of the equivalent circuits are directly computed according to the voltage – current relationships at the terminals of windings and conductors. The merits of the proposed method are that the effects in the eddy-current field, including floating conductors, internal circuits as well as displacement current in the direction of the model depth can all be taken into account in the extracted lumped parameters.

I. INTRODUCTION

In eddy-current magnetic field, the physical quantities vary sinusoidally in time and hence the magnetic field can be analyzed in the frequency domain [1]. Impedance extraction is one of the main purposes of eddy-current magnetic field computation. Complicated space distribution and time variation of the electromagnetic field can be studied based on the extracted lumped parameters in an equivalent circuit model, which is used widely to simulate electro-magnetic devices in system level [2-3]. Equivalent circuit models are used, for example, in [4] and [5] to study a wireless energy transfer system. In traditional methods the resistances are computed from the eddy-current loss; and inductances are computed from flux linkage or magnetic field energy [6]. A matrix analysis method has also been proposed by the authors to help understanding the physical meanings of these parameters [7]. The demerit of these methods is that special post-processing algorithms are required.

In this paper a simple and direct method to extract the equivalent circuit parameters of 2-diemnsional (2-D) eddy-current magnetic field is presented. It is based on the field – circuit coupled finite-element (FE) model. Its merits are that no special post-processing algorithm is needed because the same solver for the nominal problems can still be used for parameter extraction. All the effects in eddy-current field, including the floating conductors which are not the circuit ports, internal circuits which are not the ports for parameter extraction as well as the displacement current in the direction of the model depth can all be taken into account in the extracted lumped parameters. The additional ac resistances and capacitances can also be isolated from the impedances and a precise equivalent circuit can be derived.

II. PARAMETER EXTRACTION METHOD

For eddy-current magnetic field, the waveforms of all excitations are assumed sinusoidal. All equations are expressed in frequency domain. Hence all components of vector potentials, flux density, voltages and currents are represented in complex form. In a 2-D model, the magnetic field only varies on the *x*-*y* plane. It is further assumed that only $(1/p)^{\text{th}}$ of the domain needs to be solved if there is symmetry in the field distribution. In order to extract the parameters of the circuit, a FE model will be established and their basic equations in different regions will be deduced first.

After the field equations and circuit equations are available and as they can be solved in all types of excitations, the parameter extraction becomes very simple. Supposing there are totally N stranded windings and solid conductors, the equivalent circuit equation is:

$$\begin{Bmatrix} \dot{U}_{w(1)} \\ \dot{U}_{w(2)} \\ \vdots \\ \dot{U}_{w(N)} \end{Bmatrix} = \begin{Bmatrix} \dot{E}_{o(1)} \\ \dot{E}_{o(2)} \\ \vdots \\ \dot{E}_{o(N)} \end{Bmatrix} + \left\{ \begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1N} \\ R_{21} & R_{21} & \cdots & R_{2N} \\ \vdots & \vdots & \cdots & \vdots \\ R_{N1} & R_{N2} & \cdots & R_{NN} \end{bmatrix} + j\omega \begin{bmatrix} L_{11} & L_{12} & \cdots & L_{1N} \\ L_{21} & L_{21} & \cdots & L_{2N} \\ \vdots & \vdots & \cdots & \vdots \\ L_{N1} & L_{N2} & \cdots & L_{NN} \end{bmatrix} \right\} \begin{Bmatrix} \dot{I}_{w(1)} \\ \dot{I}_{w(2)} \\ \vdots \\ \dot{I}_{w(N)} \end{Bmatrix}. \quad (1)$$

Its equivalent circuit is as shown in Fig. 1.

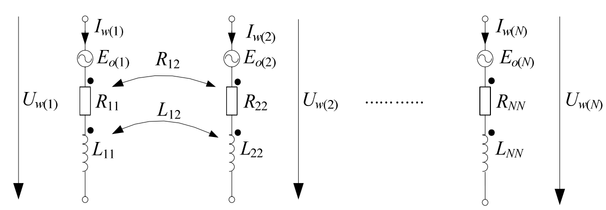

Fig. 1. The equivalent circuit of *N* windings.

The procedure to compute the parameters is:

(a) Keep all sources of current densities and internal circuits, as well as boundary conditions; set all sources of stranded windings and solid conductors to be zero; compute the eddy-current field, then compute the back emf $\dot{E}_{o(i)}$ (i = 1, 2, …, N) in each stranded windings and solid conductors.

(b) Set all non-zero-value boundary conditions to zero;

set all internal sources to zero; set a unit current source only in the i^{th} stranded winding or solid conductor; set the current sources in all other stranded windings or solid conductors to zero; compute the eddy-current magnetic field – circuit coupled problem and then compute the voltages $\dot{U}_{w(i)}$ (i = 1, 2, …, N) in each windings. The resistance is:

$$R_{ij} = \mathrm{Re}\left(\frac{\dot{U}_{w(j)}}{\dot{I}_{w(i)}}\right). \qquad (j = 1, 2, \ldots, N) \qquad (2)$$

The inductance is:

$$L_{ij} = \frac{1}{\omega}\mathrm{Im}\left(\frac{\dot{U}_{w(j)}}{\dot{I}_{w(i)}}\right). \qquad (j = 1, 2, \ldots, N) \qquad (3)$$

When each FE field is computed, the coefficient matrix of the system algebraic equation is kept the same; only the right hand side changes and only a multi right hand side (RHS) problem needs to be solved. By using the multi-RHS algebraic solvers, the computing time required to extract the parameters can be greatly reduced.

The following formulation will reveal the relationship among the parameters. If there is only excitation in the i^{th} stranded winding, the voltage of winding i can be obtained as follows:

$$\dot{U}_{w(i)} = \mathrm{Im}\left[-\frac{\frac{l}{S}\iint_{\Omega_i}\omega\dot{A}\,d\Omega}{\dot{I}_{w(i)}}\right]\dot{I}_{w(i)} + j\omega\,\mathrm{Re}\left[\frac{\frac{l}{S}\iint_{\Omega_i}\dot{A}\,d\Omega}{\dot{I}_{w(i)}}\right]\dot{I}_{w(i)} + \frac{1}{G_{w(i)} + j\omega C_{w(i)}}\dot{I}_{w(i)} \qquad (4)$$

The total voltage of the winding i is:

$$\dot{U}_{w(i)} = \dot{E}_{o(i)} + \sum_{j=1}^{N} R_{w(ij)}\dot{I}_{w(j)} + j\omega\sum_{j=1}^{N} L_{w(jj)}\dot{I}_{w(j)} + \frac{1}{G_{w(i)} + j\omega C_{w(i)}}\dot{I}_{w(i)} \ . \qquad (5)$$

From (5) a precise equivalent circuit can be obtained. Its equivalent circuit is shown in Fig. 2. Because the dc conductance $G_{w(i)}$ and capacitance $C_{w(i)}$ can be computed easily. $R_{w(ii)}$ and $L_{w(ii)}$ can also be isolated if necessary.

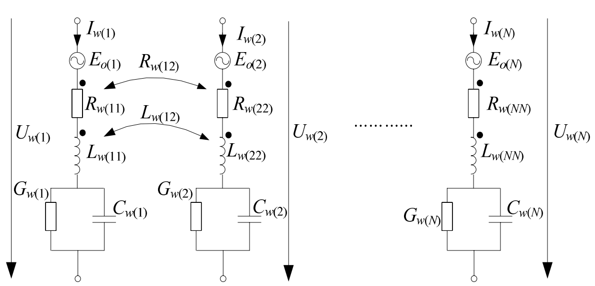

Fig. 2. The equivalent circuit of N windings with isolated parameters.

III. TEST CASE

The computation of the impedance between the two conductors of a coaxial cable as shown in Fig. 3 is used as an example. The space between the cylindrical conductors is filled with three layers of dielectrics. ε_{r1} = 11.9 and ε_{r2} = 2.25. The dimensions are: d_1 = 6.858 mm, d_2 = 7.874 mm, d_3 = 19.177 mm, d_4 = 25.396 mm, and d_5 = 25.65 mm. The computed results of resistances and inductances versus frequency are shown in Table I, which shows that the solutions using the proposed method are the same as those obtained by traditional method. The analytical solution of the dc resistance is 0.0021599Ω/m, which is the same as that obtained by the numerical method at low frequency; the analytical solution of the inductance between the two conductors when the currents are only located on the surfaces of the conductors is 0.262 uH/m, which is the same as that by the numerical method at high frequency, because at high frequency, the currents flow on the surfaces of the conductors.

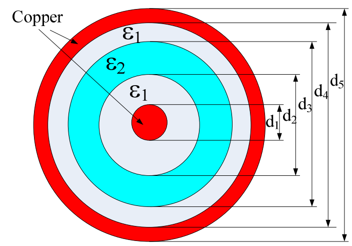

Fig. 3. The cross section of a coaxial cable.

TABLE I
THE COMPUTED RESULTS OF THE IMPEDANCE

Frequency	Resistance (Ω/m)		Inductance (uH/m)	
	Traditional method	Proposed method	Traditional method	Proposed method
1 Hz	0.0021847	0.0021609	0.31247	0.31193
10 Hz	0.0021847	0.0021609	0.31246	0.31250
100 Hz	0.0021854	0.0021616	0.31243	0.31190
1kHz	0.0022472	0.0022238	0.30919	0.30916
10 kHz	0.0030574	0.0032842	0.28168	0.27780
100 kHz	0.0057164	0.0044940	0.26858	0.26234
1 MHz	0.0154310	0.0153540	0.26423	0.26428

IV. REFERENCES

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